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Upper and Lower Control Limits for Means in Cases of Non-normal Variation and EWMA Model

Ketki Kulkarni

Associate Prof. BSSS Institute of Advanced Studies, Bhopal. E-mail: ketki091979@gmail.com

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Abstract: The presence of non-normality and EWMA model have a significant effect on the control limits of X-control chart. The effect of non-normality under EWMA model on the charts, we conclude that non-normality is usually not a problem for subgroup sizes of four or more but there is substantial effect of EWMA constant on upper and lower control limits.

Keywords: Upper and lower control limits, control charts, EWMA model, non-normality.

INTRODUCTION

Probably no statistical method is as characteristics of quality control and industrial statistics as the control chart. Its uses are many and varies, including for example: studies of process capability (Clifford (1971)), measurement capability studies (Wernimant (1951)), presentation of the results of designed experiments (Ott (1967)), and acceptance sampling for process parameters (Freund (1957)), as well as in the traditional sense of process control (Knowler (1946)). In many applications the chart is applied without knowledge of the shape of the underlying distribution of individuals. Indeed, it is often stated that the distribution of a process is used to establish control prior to determining the distribution of the underlying process. When an $\bar{X}$-control chart having 3σ limits is employed with a process which is normally distributed the type-I risk (i.e., risk of a point falling outside the limits when the process is in control) associated with these control limits is 0.003. For other underlying distributions, however this may not be the case of particular interest, therefore, in application of control charts as well as in most practical applications of the central limit theorem is the rate at which the distribution of sample means approaches the normal distribution. Shewhart (1931) has shown empirically through

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the use of Shewhart's bowl that the standard control chart limits are approximately correct for the right triangular and rectangular distributions. Also, a set of tables of 3? control limit factors for non-normal distributions has been presented by Burr (1967). From his study of the effect of non-normality on these factors, he concludes that, "----we can use the ordinary normal curve control chart constants unless the population is markedly non-normal. When it is, the tables provide guidance on what constant to use "(Burr (1967)). Burr's value were derived for various members of the Burr family of distribution from the expected values of the range for those distributions. They provide an approximation to the limits for other non-normal distributions based on the associated coefficients of skewness and kurtosis. The exact probability of exceeding these control limits when the process is in control, however, remains unknown. Shewhart (1931) emphasizes that most distributions exhibiting control have been found to be in close neighborhood of normality to be fitted by the first two terms of the Gram-Charlier series. But it seems reasonable and also necessary sometimes to consider a better form with terms including upto that in β_1 of the Edgeworth series.

The control limits for $\bar{X}$-chart and σ-chart calculated on the basis of normal population may be seriously affected particularly in cases of variations showing significant departures of β_1 and β_2 from their respective normal theory values (Delaporte (1951)) recommends utilization of such estimates in the formulae for the β-coefficients of the sample characteristics concerned in order to choose a suitable Person curve to represent its frequency distribution. Working with the standardize variable $x = \{X\text{-}E(x)\}/\sigma_X$, in a significantly non-normal situation, one can thus obtain a satisfactory Pearson curve for the distribution of $\bar{x}$ and calculate upper and lower limits for conventional probability levels. The preparation of tables of values of mean $\bar{x}$, for a number of probability levels and for suitable ranges of values of β_1 and β_2 is worthwhile, as Delaporte suggests, but the test seems to be formidable by his method of approach. The tables of the probability integrals of the Pearson curves, which will be required for the purpose, are available only for a few of the curves in a form one gets in the case of the normal probability function. Although formulae are there for calculating the desired fractal for all other cases, they do not appear to help much in the preparation of the proposed table.

Pearson and Please (1975) used the results of extensive sampling experiments on theoretical and empirical populations to prepare a series of charts in which the robustness of the one and two-sample "student's t" statistics, the sample variance, and the variance ration are displayed as functions of the skewness and kurtosis of the populations. Andrews *et. al.*

(1972), Huber (1972), Hampel (1974), and Hogg (1974) reviewed robust estimation and compared many estimators of a location parameter with respect to their robustness and efficiency. Stigler (1977) examined the performance of some robust estimators of location when applied to some historical data and concluded that the sample mean compares favorably but the light "trimming" of the outer order statistics from the computation of the mean greatly improves performances.

Gayen (1951) had considered the robustness of both the sample correlation coefficient r and of Fisher's Z transformation of departures from bivariate normality. When the population correlation coefficient ρ is zero and, in particular, when the variables are independent, the distribution of *r* is robust even for small sample sizes, however, for large values of ρ, the departures from normality are appreciable. The Z-transformation under non-normal parent population, remains asymptotically normal but the approach to normality is somewhat tardy. The mean and the variance of Z are unaffected by the bivariate parent population, but the effects of departures from mesokurtosis may be considerable. Although the mean of Z slowly approaches its normal value as n increases, its variance is sensitive to the shape of the parent population even in large samples.

Subsequent studies dealing with the distributions of r and z (and various other transformations of r) include: Harley (1956), Kowalski (1972), Gajjar and Kocherlakota (1978), Kocherlakota and Gajjar (1980), Kocherlakota and Kocherlakota (1985), Kocherlakota and Singh (1982) and Srivastava and Lee (1984, 1985). From these studies, the results on robustness of various tests, particularly the z-test for testing ρ = 0 collaborate the earlier findings and can be summarized as follows: The z-test and various other tests based on the classical sample correlation coefficient r, are robust to small departure from normality. For more than moderate departures, however, he type-I errors of these tests, including the *z*-test, are not at all robust and are in fact considerably larger than their advertised values. Tiku (1986) has developed a parametric test for H_0: ρ = 0 which has much better robustness properties for situations more commonly encountered in practice.

Finally, we mention briefly the problem of testing normality. Sometimes, it is common to test the observed moment ratio $(b_1)^{1/2}$ and b_2 against their distributions, given the hypothesis of normality, and these are occasionally referred to tests of normality. This is, however, a very loose description of normality, and such tests are better called tests of skewness and kurtosis respectively. Geary (1947b) developed and investigated an alternative test of kurtosis based on the ratio of sample mean deviation to the standard deviation. Gastwirth and Owens (1977) showed that Geary's test may be superior to b_2 in detecting symmetric departures from normality.

Kac *et al.* (1955) discussed the distribution of D_n and w^2 in testing normality when the two parameters (μ, σ^2) are estimated from sample by ($\bar{X}$, S^2). The limiting distributions of these statistics are parameter-free, but are not obtained explicitly. Some sampling experiments are reported which give empirical estimates of these probabilities. Lilliefors (1967) used extensive sampling experiments to compute critical values of Kolmogorov-Smirnov D_n statistic in testing normality. Shapiro and Wilk (1965) proposed a new criterion, called W-test, for testing normality which is based on the regression of the order statistics upon their expected values. They carried out extensive sampling experiments to evaluate its distribution. Shapiro *et al.* (1968) and Stephens (1974) made power comparisons of various tests for normality, using extensive sampling experiments, and showed that W-test is usually somewhat superior to other tests in their study. Shapiro and Francia (1972) developed a simplified approximation of W-test for large samples, whose consistency was established by Sarkadi (1975) (see, also Weisberg (1974)) Doksum *et al.* (1977) discussed plots and tests for assessing symmetry, some of which compared favorably in power with the W-test. Moore (1971) developed chi-square statistics with random cell boundaries for testing normality when fully efficient maximum likelihood estimators are used to determine boundaries with estimated equal probabilities in classes (see, also Mardia (1980)).

The investigations on robustness, as summarized heretofore, have as their aim the recognition of the range of validity of the standard normal theory procedures. However, it is often difficult in practice to decide whether the standard procedures are likely to be approximately valid or misleading. One common approach to the non-fulfillment of normality assumption has been to seek a transformation which will bring the observations to the normal form, so that standard normal theory procedures may be applied to the transformed data. The early investigation in this field were carried out by Bartlett (1947). Hoyle (1973) gives an excellent review and a compendium of references to earlier works.

Another alternative approach to non-normality is a radical one. Instead of holding to standard normal theory methods, either because they are robust and approximately valid in non-normal cases, or by transforming the observations to make them approximately valid, they are entirely abandoned and the problem is approached afresh. The intent is to find statistical procedures which remain valid for a wide class of parent distributions, say for all continuous distributions. If such procedures are developed, they will necessarily be valid for normal distributions, and their robustness will be precise and assured. Such procedures are called distribution-free methods since they are not dependent on a given distribution (such as the normal), but will work for a wide range of different distributions. They are also called non-parametric methods because their null hypothesis is not concerned with specific parameters (such as the mean or variance), but only with the distribution of the variates. In recent years, distribution-free or non-parametric method have become quite popular because they are readily computable and permit freedom from worry about the classical assumptions of the standard normal theory. It should however, be pointed out that in cases where classical assumptions hold entirely or even approximately, the analogous standard normal procedures are generally more efficiently for detecting departures from the null hypothesis.

In general, the process parameters μ and σ must be estimated from process data so that the control limits can be determined. Of course, with more data, the control limits can be estimated more precisely. One risk of using too few data points is that the control limits will be poorly estimated. If σ is underestimated, then the limits will be too narrow and there will be too many false alarms. If σ is overestimated then the limits will be too wide and the chart will rarely signal even for moderate shifts. A second risk of having n, the size of the preliminary sample, too small is that it will be difficult to assess whether the process was in control when the data were collected. If n is sufficiently small then it is impossible to have a point outside the control limits on the retrospective $\bar{X}$-chart.

There are many industrial situations where it does not make sense to periodically collect a sample containing more than one observation since consecutive observations differ only due to measurement error. In these situations, individual observations are periodically drawn from the process. Individual observation control charts are often used to monitor chemical processes where, for example, the measurement is temperature or concentration. Also, these charts are used to monitor processes under engineering feedback process control, an area of increased attention recently (see, e.g., Box and Kramer (1992), and Montgomery *et al.* (1994)). Individual observation control charts may be appropriate when automated inspection and measurement technology is used and every unit is measured. They are also appropriate when the production is rate low or the cost of measurement is high. In such a situation one can apply the exponential weighted moving average (EWMA) chart. The EWMA chart introduced by Roberts (1959), may be more difficult to interpret than an X-chart but is more effective in detecting small shifts in the process mean (see, Hunter (1986), Robinson and Ho (1978), Crowder (1987a, 1987c, 1989), and Lucas and Saccucci (1990)). Saccucci and Lucas (1990) give a computer program for computing the average run length (ARL) associated with the EWMA and the combined X and EWMA charts. Hunter (1986) points out that the EWMA chart for sample average can be nicely graphed simultaneously with the Shewhart chart to enable easier interpretation.

EWMA charts have been developed to detect shifts in the process variability. Ng and Case (1989) constructed several EWMA charts including one where the sample statistic is the weighted average of moving ranges. Wortham and Ringer (1971) and Sweet (1986) suggest using two EWMA charts, one for detecting mean shifts and the other for detecting variance shifts. Macgregor and Harris (1993) propose EWMA charts for controlling variance that also could be used for individual observations. CUSUM charts are another important alternative for individual observations. Lucas and Saccucci (1990) compare the CUSUM and EWMA charts in monitoring the process mean and conclude, "The properties of control schemes are so close that we had to consider such things as the steady state distribution (of the run length, as apposed to simply the mean) to differentiate between them…We feel that other criteria, such as training costs could be used to determine which control scheme is implemented".

Since the effectiveness of CUSUM and EWMA charts are so close, it is clear that one can construct a combination procedure of CUSUM chart and X chart that meets (or even slightly exceeds) the run length performance of the X and EWMA chart. Following the advice in Lucas and Saccucci (1990) we consider the EWMA chart in this study since the EWMA chart is somewhat familiar in our application area. The EWMA is a useful monitoring tool that has the following main properties:

(i) It allows us to monitor μ and σ on one chart.

(ii) Increases and /or decreases in either of the process parameters can be directly identified.

(iii) It is useful graphical diagnostic tool.

(iv) It has good ARL properties for simultaneous changes in the process mean and standard deviation and for large increases in σ. (when there are decreases in the variance with respect to the mean it is not as efficient as simultaneously using the X and EWMA charts)

(v) It allows the placement of specification limits on the chart.

(vi) It may be viewed as smoothed tolerance limits.

(vii) In situations where it is possible to rank order the observations before measurements are actually taken, the EWMA chart requires only two measurements.

The aim of the present paper is to study the problem of setting up control limits for means in cases of non-normal variation and EWMA model. The non-normal distribution has been represented by the first four terms of an

Edgeworth series. The values of standardized cumulant $\lambda_3 = \sqrt{\beta_1}$, and $\lambda_4 = \beta_2 - 3$ considered are within Barton and Dennis (1952) limits, which means that for such values the population is positive definite and unimodal. For various non-normal populations and different values of λ under EWMA model, the values of upper and lower control limits are tabulated and compared with those of the normal population.

3.2. Modeling and Monitoring EWMA Control Chart

The EWMA control chart is based on the exponentially weighted moving average which is defined as

$$w_t = \lambda x_t + (1-\lambda) w_{t-1} \qquad (3.2.1)$$

where x_t is the observation at time t, λ is a smoothing constant $(0 < \lambda \le 1)$, and the starting value, w_0 is set equal to the in-control process mean. The EWMA control chart signals if the EWMA statistic, w_t, falls outside the control limits. The sequence of w_t is called EWMA

$$w_t = \lambda x_t + (1-\lambda)\,[\lambda x_{t-1} + (1-\lambda)\, w_{t-2})] \qquad (3.2.2)$$

$$= \lambda x_t + \lambda(1-\lambda)\, x_{t-1}) + (1-\lambda)^2\, w_{t-2}$$

$$= \lambda x_t + \lambda(1-\lambda) x_{t-1} + (1-\lambda)^2\, [\lambda x_{t-2} + (1-\lambda)\, w_{t-3}]$$

$$= \lambda x_t + \lambda(1-\lambda)\, x_{t-1} + \lambda(1-\lambda)^2\, x_{t-2} + (1-\lambda)^3\, w_{t-3}$$

$$= \lambda w_t + \lambda(1-\lambda) x_{t-1} + \lambda(1-\lambda)^2\, x_{t-2} + \ldots + \lambda(1-\lambda)^{t-1}\, x_1 + (1-\lambda)^t\, \mu,$$

where $w_0 = \mu$. Here we compute the mean and variance of w_t.

$$E(w_t) = \mu[\lambda + \lambda(1-\lambda) + \lambda(1-\lambda)^2] + ---+(1-\lambda)^t$$

$$= \frac{\mu\lambda[1-(1-\lambda)^t]}{1-(1-\lambda)} + \mu(1-\lambda)^t$$

$$= \mu$$

and $Var(w_t) = \dfrac{\sigma^2}{n}[\lambda^2 + \{\lambda(1-\lambda)\}^2 + \{\lambda(1-\lambda)^2\}^2 + --- + \{\lambda(1-\lambda)^{t-1}\}^2]$

$$= \frac{\sigma^2}{n}\lambda^2[1 + \theta + \theta^2 + --- + \theta^{t-1}]$$

where $$\theta = (1-\lambda)^2$$

$$Var(w_t) = \frac{\sigma^2}{n}\lambda^2 \frac{[1-(1-\lambda)^{2t}]}{(2-\lambda)}, \text{ since } (1-\lambda)^{2t} = 0$$

$$= \frac{\sigma^2}{n} D^2$$

where $$D^2 = \left(\frac{\lambda}{2-\lambda}\right)$$

3.3. The effect of Non-Normality and EWMA model on control limits for means

We consider the frequency function for the quality characteristics to be represented by the Edgeworth form of Type A series then the probability integral for it has been given by Cornish and Fisher in terms of the normal probability levels. Let ξ denote the variable for the quality characteristic in standardized form i.e., having zero mean and unit standard deviation and x the standard normal variate. The p percent probability levels of the standard normal variable $x = (w_t - \mu)/\sigma$ by the expression

$$\begin{aligned}\xi = x\frac{1}{6}\sqrt{\beta_1}(x^2-1) + \frac{1}{24}(\beta_2-3)(x^3-3x) - \frac{1}{36}\beta_1(2x^3-5x) + \frac{1}{120}\lambda_5(x^4-6x^2+3)\\
-\frac{1}{24}\lambda_3\lambda_4(x^4-5x^2+2) + \frac{1}{324}\lambda_3^2(12x^4-53x^2+17)\\
+\frac{1}{720}(x^5-10x^3+15x) - \frac{1}{384}\lambda_4^2(3x^5-24x^3+29x)\\
-\frac{1}{180}\lambda_3\lambda_5(2x^5-17x^3+21x) + \frac{1}{288}\lambda_3^2\lambda_4(14x^5-103x^3+107x)\\
-\frac{1}{7776}\lambda_3^4(252x^5-1688x^3+151x),\end{aligned}$$

(3.3.1)

where, $\lambda_3 = \sqrt{\beta_1}, \lambda_4 = (\beta_2-3), \lambda_5$ and λ_6 are standardised cumulants defined

by $\lambda_i = \left(\frac{k_i}{k^{\frac{1}{2}}_2}\right)$ of the distribution of a statistic we should find out the first few

λ-coefficients of the statistic for the determinations of the probability integral in terms of the normal deviate x by the equation (3.3.1). In the case of control chart for average ξ we should therefore, obtain the standardized cumulants of ξ and substitute them in the above expressions to obtain the necessary control limits.

When the basic variable ξ follows the Edgeworth series the mean ξ' also follows the same law but with different values for standardized cumulants whose expressions are already known. If we consider the moderately non-normal populations, terms upto that in β_1 will provide good approximation. This is particularly so for the distribution of mean even in cases where the basic populations may need more terms of the Edgeworth series for a satisfactory representation. Owing to the fact that standardized cumulants of the distributions of the mean are of order n^{-i}, where n is the size of the sample and $i = \frac{1}{2}, 1, \frac{3}{2}, ...$ When we stick to the first four moments and EWMA model, we obtain the simpler expression, by neglecting powers of λ_4, λ_3^2, and terms of order higher than those in λ_4 and λ_3^2 such as:

$$\xi' = x + \frac{1}{6}\lambda_3 D(x^2 - 1) + \frac{1}{24}\lambda_4(x^3 - 3x) - \frac{1}{36}\lambda_3^2 D^2(2x^3 - 5x)$$

$$= x + \lambda_3 D\, M_3(x) + \lambda_4 D^2 M_4(x) + \lambda_3^2 D^2 M_{33}(x), \qquad (3.3.2)$$

where $$M_3(x) = \frac{1}{6}(x^2 - 1),\ M_4(x) = \frac{1}{24}(x^3 - 3x),$$

and $$M_{33}(x) = -\frac{1}{36}(2x^3 - 5x)$$

In fact, β_1 and β_2 for the basic variable may even be considerably large, for those of the statistic (here, mean) is less in magnitude. We know for the mean

$$\beta_1(\xi') = \frac{\beta_1(\xi'_1)}{n} \quad and \quad \beta_2(\xi') = 3 + \frac{\beta_2(\xi'-3)}{n}$$

and note that the probability integrals for non-normal variable ξ and EWMA model

$$\int_{-\infty}^{\xi_1} p(\xi)d\xi = \int_{\xi_2}^{\infty} p(\xi)\, d\xi = 0.005$$

in which ξ_1 and ξ_2 is given by

$$\bar{\xi} = \bar{x} + \frac{\lambda_3}{D} M_3(\bar{x}) + \frac{\lambda_4}{D^2} M_4(\bar{x}) + \frac{\lambda_3^2}{D^2} M_{33}(\bar{x}), \qquad (3.3.3)$$

where $\bar{x} = \pm\, 2.576\, D.$

3.4. Numerical Illustration and Discussion of the Results.

For various non-normal population with various non-normality parameters (λ_3, λ_4) and EWMA constant λ, the values of upper and lower control limits are given in Table 3.1 to Table 3.5. If λ =1 we get the tabulated values of Gayen (1957) which is shown in Table (3.5). The effect of non-normality under EWMA model on the charts and we conclude that non-normality is usually not a problem for subgroup sizes of four or more but there is substantial effect of EWMA constant on upper and lower control limits. From Table 3.1 to Table 3.5 it is evident that upper control limits for λ = 0.2, 0.4, 0.6, 0.8, 1.0 and (λ_3, λ_4) = (-0.60, -0.50) are 1.5886, 1.3869, 1.3906, 1.4690, 1.6042, respectively, whereas lower control limit for above constants are -1.4169, - 1.6504, - 1.9538, -2.3076 and -2.7314 respectively. By comparing entries of Table 3.5 for =1 with Table 3.1 to Table 3.4 when λ = 0.2, 0.4, 0.6 and 0.8 one can easily see that the effect of EWMA model is quite series on the lower and upper control limits. Thus, we see that either the values of the upper limits or lower limits will serve the purpose. Just to avoid negative signs it is advisable to tabulate values of upper limits, since lower limits

Table 3.1: Upper and Lower Control Limits for Non-Normal Population Under EWMA Model (λ =0.2)

$\lambda_3 \rightarrow$ $\lambda_4 \downarrow$	*0.1*	*0.4*	*0.7*	*1*	*-1*	*-0.6*	*-0.3*	*0*
-0.5	-1.2439	-1.3969	-1.6861	-2.1116	-1.8488	-1.4169	-1.2519	-1.2232
	1.2317	1.3058	1.5162	1.8629	2.1257	1.5886	1.3448	1.2233
-0.35	-1.1345	-1.2875	-1.5767	-2.0023	-1.7394	-1.3075	-1.1425	-1.1138
	1.1181	1.1922	1.4026	1.7493	2.0121	1.4751	1.2312	1.1139
-0.15	-0.9887	-1.1417	-1.4309	-1.8564	-1.5936	-1.1617	-0.9967	-0.9680
	0.9667	1.0408	1.2512	1.5979	1.8607	1.3236	1.0798	0.9681
0	-0.8793	-1.0323	-1.3215	-1.7471	-1.4842	-1.0523	-0.8873	-0.8586
	0.8531	0.9273	1.1377	1.4843	1.7472	1.2101	0.9663	0.8587
0.05	-0.8429	-0.9958	-1.2851	-1.7106	-1.4478	-1.0158	-0.8509	-0.8221
	0.8153	0.8894	1.0998	1.4465	1.7093	1.1722	0.9284	0.8222
0.35	-0.6241	-0.7771	-1.0663	-1.4919	-1.2290	-0.7971	-0.6321	-0.6034
	0.5882	0.6623	0.8727	1.2194	1.4822	0.9451	0.7013	0.6035
0.5	-0.5147	-0.6677	-0.9570	-1.3825	-1.1197	-0.6877	-0.5227	-0.4940
	0.4746	0.5487	0.7591	1.1058	1.3686	0.8316	0.5877	0.4941
0.9	-0.2231	-0.3761	-0.6653	-1.0908	-0.8280	-0.3961	-0.2311	-0.2024
	0.1718	0.2459	0.4563	0.8030	1.0658	0.5287	0.2849	0.2025

Table 3.2: Upper and Lower Control Limits for Non-Normal Population Under EWMA Model (λ = 0.4)

$\lambda_3 \rightarrow$ $\lambda_4 \downarrow$	*0.1*	*0.4*	*0.7*	*1*	*-1*	*-0.6*	*-0.3*	*0*
-0.5	-1.4124	-1.3827	-1.3963	-1.4532	-1.8924	-1.6504	-1.5196	-1.4320
	1.4564	1.5584	1.7037	1.8924	1.4532	1.3869	1.3878	1.4320
-0.35	-1.3692	-1.3395	-1.3531	-1.4100	-1.8492	-1.6072	-1.4764	-1.3888
	1.4132	1.5152	1.6605	1.8492	1.4100	1.3437	1.3446	1.3888
-0.15	-1.3116	-1.2819	-1.2955	-1.3524	-1.7916	-1.5496	-1.4188	-1.3312
	1.3556	1.4576	1.6029	1.7916	1.3524	1.2861	1.2870	1.3312
0	-1.2684	-1.2387	-1.2523	-1.3092	-1.7484	-1.5064	-1.3756	-1.2880
	1.3124	1.4144	1.5597	1.7484	1.3092	1.2429	1.2438	1.2880
0.05	-1.2540	-1.2243	-1.2379	-1.2948	-1.7340	-1.4920	-1.3612	-1.2736
	1.2980	1.4000	1.5453	1.7340	1.2948	1.2285	1.2294	1.2736
0.35	-1.1676	-1.1379	-1.1515	-1.2084	-1.6476	-1.4056	-1.2748	-1.1872
	1.2116	1.3136	1.4589	1.6476	1.2084	1.1421	1.1430	1.1872
0.5	-1.1244	-1.0947	-1.1083	-1.1652	-1.6044	-1.3624	-1.2316	-1.1440
	1.1684	1.2704	1.4157	1.6044	1.1652	1.0989	1.0998	1.1440
0.9	-1.0092	-0.9795	-0.9931	-1.0500	-1.4892	-1.2472	-1.1164	-1.0288
	1.0532	1.1552	1.3005	1.4892	1.0500	0.9837	0.9846	1.0288

Table 3.3: Upper and Lower Control Limits for Non-Normal Population Under EWMA Model (=0.6)

$\lambda_3 \rightarrow$ $\lambda_4 \downarrow$	*0.1*	*0.4*	*0.7*	*1*	*-1*	*-0.6*	*-0.3*	*0*
-0.5	-1.6515	-1.4995	-1.3339	-1.1547	-2.0935	-1.9538	-1.8333	-1.6992
	1.7454	1.8750	1.9910	2.0935	1.1547	1.3906	1.5517	1.6992
-0.35	-1.6477	-1.4956	-1.3300	-1.1509	-2.0896	-1.9500	-1.8294	-1.6954
	1.7416	1.8711	1.9871	2.0896	1.1509	1.3867	1.5478	1.6954
-0.15	-1.6426	-1.4905	-1.3249	-1.1458	-2.0845	-1.9448	-1.8243	-1.6902
	1.7364	1.8660	1.9820	2.0845	1.1458	1.3816	1.5427	1.6902
0	-1.6387	-1.4866	-1.3210	-1.1419	-2.0807	-1.9410	-1.8205	-1.6864
	1.7326	1.8621	1.9782	2.0807	1.1419	1.3777	1.5388	1.6864
0.05	-1.6374	-1.4853	-1.3197	-1.1406	-2.0794	-1.9397	-1.8192	-1.6851
	1.7313	1.8608	1.9769	2.0794	1.1406	1.3764	1.5375	1.6851
0.35	-1.6297	-1.4776	-1.3120	-1.1329	-2.0717	-1.9320	-1.8115	-1.6774
	1.7236	1.8531	1.9692	2.0717	1.1329	1.3687	1.5298	1.6774
0.5	-1.6259	-1.4738	-1.3082	-1.1291	-2.0678	-1.9281	-1.8076	-1.6736
	1.7198	1.8493	1.9653	2.0678	1.1291	1.3649	1.5260	1.6736
0.9	-1.6156	-1.4635	-1.2979	-1.1188	-2.0576	-1.9179	-1.7974	-1.6633
	1.7095	1.8390	1.9551	2.0576	1.1188	1.3546	1.5157	1.6633

Table 3.4: Upper and Lower Control Limits for Non-Normal Population Under EWMA Model (=0.8)

$\lambda_3 \rightarrow$ $\lambda_4 \downarrow$	*0.1*	*0.4*	*0.7*	*1*	*-1*	*-0.6*	*-0.3*	*0*
-0.5	-1.9364	-1.6762	-1.3553	-0.9737	-2.3713	-2.3076	-2.1890	-2.0097
	2.0762	2.2353	2.3337	2.3713	0.9737	1.4690	1.7697	2.0097
-0.35	-1.9645	-1.7043	-1.3834	-1.0017	-2.3994	-2.3357	-2.2171	-2.0378
	2.1043	2.2634	2.3617	2.3994	1.0017	1.4971	1.7978	2.0378
-0.15	-2.0020	-1.7417	-1.4208	-1.0392	-2.4369	-2.3731	-2.2545	-2.0752
	2.1417	2.3008	2.3992	2.4369	1.0392	1.5345	1.8352	2.0752
0	-2.0300	-1.7698	-1.4489	-1.0673	-2.4649	-2.4012	-2.2826	-2.1033
	2.1698	2.3289	2.4273	2.4649	1.0673	1.5626	1.8633	2.1033
0.05	-2.0394	-1.7792	-1.4582	-1.0766	-2.4743	-2.4106	-2.2920	-2.1127
	2.1792	2.3382	2.4366	2.4743	1.0766	1.5720	1.8727	2.1127
0.5	-2.1236	-1.8634	-1.5425	-1.1609	-2.5305	-2.4667	-2.3481	-2.1688
	2.2634	2.4225	2.5209	2.5585	1.1328	1.6281	1.9288	2.1688
0.35	-2.0956	-1.8353	-1.5144	-1.1328	-2.5585	-2.4948	-2.3762	-2.1969
	2.2353	2.3944	2.4928	2.5305	1.1609	1.6562	1.9569	2.1969
0.9	-2.1985	-1.9383	-1.6174	-1.2357	-2.6334	-2.5697	-2.4511	-2.2718
	2.3383	2.4974	2.5957	2.6334	1.2357	1.7311	2.0318	2.2718

Table 3.5: Upper and Lower Control Limits for Non-Normal Population Under EWMA Model (=1)

$\lambda_3 \rightarrow$ $\lambda_4 \downarrow$	0.1	0.4	0.7	1	-1	-0.6	-0.3	0
-0.5	-2.2811	-1.9105	-1.4334	-0.8497	-2.7283	-2.7314	-2.6094	-2.3809
	2.4689	2.6619	2.7484	2.7283	0.8497	1.6042	2.0458	2.3809
-0.35	-2.3396	-1.9690	-1.4919	-0.9082	-2.7868	-2.7899	-2.6679	-2.4394
	2.5274	2.7204	2.8069	2.7868	0.9082	1.6628	2.1044	2.4394
-0.15	-2.4176	-2.0470	-1.5699	-0.9863	-2.8649	-2.8680	-2.7460	-2.5175
	2.6055	2.7985	2.8849	2.8649	0.9863	1.7408	2.1824	2.5175
0	-2.4762	-2.1056	-1.6285	-1.0448	-2.9234	-2.9265	-2.8045	-2.5760
	2.6640	2.8570	2.9435	2.9234	1.0448	1.7993	2.2409	2.5760
0.05	-2.4957	-2.1251	-1.6480	-1.0643	-2.9429	-2.9460	-2.8240	-2.5955
	2.6835	2.8765	2.9630	2.9429	1.0643	1.8188	2.2604	2.5955
0.35	-2.6127	-2.2421	-1.7650	-1.1814	-3.0600	-3.0631	-2.9411	-2.7126
	2.8006	2.9936	3.0800	3.0600	1.1814	1.9359	2.3775	2.7126
0.5	-2.6713	-2.3007	-1.8236	-1.2399	-3.1185	-3.1216	-2.9996	-2.7711
	2.8591	3.0521	3.1386	3.1185	1.2399	1.9944	2.4360	2.7711
0.9	-2.8273	-2.4568	-1.9796	-1.3960	-3.2746	-3.2777	-3.1557	-2.9272
	3.0152	3.2082	3.2947	3.2746	1.3960	2.1505	2.5921	2.9272

have always negative values. An unstable process can lead to a seemingly non-normal distribution and EWMA model. If for example, the process shifted upward after two-thirds of the data were collected, then there would be skewed to the right. In such cases, a data transformation would be inappropriate. It is thus important that the data be taken from a stable process. When the process is believed to be stable, check for non-normality and EWMA model by looking at a histogram and a normality probability curve. What is important to note here is, that the presence of non-normality and EWMA model have a significant effect on the control limits of X-control chart.

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